ASSUMPTIONS

In this section we shall make several specializing assumptions which will lead to certain particular cases of the general mixture theory described above. We shall first assume that the stress in all constituents is sufficiently high so that a hydrodynamic stress state prevails. That is

$$\mathbf{T}_\alpha = -P_\alpha \mathbf{1} \tag{21}$$

where P_α is called the partial pressure for the constituent S_α. We may also define a crystal pressure, $\widetilde{P}_\alpha$, given by

$$\widetilde{\mathbf{T}}_\alpha = -\widetilde{P}_\alpha \mathbf{1} \tag{22}$$

Then from Equation (18) we have

$$P_\alpha = n_\alpha \widetilde{P}_\alpha \tag{23}$$

Next, we restrict further consideration to plane shock waves running into undisturbed material. The constituent density leading the shock, ρ_α^+, will thus be the initial density, ρ_{α_0}. Also we need only consider the components of velocity and heat flux normal to the planar shock surface. These will be designated v_α and h_α. Further, we let the velocity, internal energy, pressure, and heat flux leading the shock vanish. That is

$$v_\alpha^+ = \epsilon_\alpha^+ = P_\alpha^+ = h_\alpha^+ = 0, \qquad \alpha = 1, 2, \ldots, k \tag{24}$$

These assumptions are made solely for the sake of convenience. The resulting equations are somewhat more cumbersome if the above assumptions are not made, but they offer no more real difficulty in derivation.

Using the above assumptions, the jump equations [Equations (10), (11), and (12)] can be written as

$$\rho_\alpha^-(U - v_\alpha^-) = \rho_{\alpha_0} U + \underset{\sigma}{\hat{c}_\alpha} \tag{25}$$

$$P_\alpha^- + \underset{\sigma}{\hat{m}_\alpha} = \rho_\alpha^-(U - v_\alpha^-)\, v_\alpha^- \tag{26}$$

$$P_\alpha^- v_\alpha^- + \underset{\sigma}{\hat{\epsilon}_\alpha} = \rho_\alpha^- \left(U - v_\alpha^-\right)\left(\epsilon_\alpha^- + \frac{1}{2}\,(v_\alpha^-)^2\right) + h_\alpha^- \tag{27}$$

Here, we have used U and $\underset{\sigma}{\hat{m}_\alpha}$ to designate the components of $\mathbf{U}$ and $\underset{\sigma}{\hat{\mathbf{m}}_\alpha}$ normal to the shock surface. These three equations are precisely the usual jump relations for a single material except for the supply terms $\underset{\sigma}{\hat{c}_\alpha}$, $\underset{\sigma}{\hat{m}_\alpha}$, and $\underset{\sigma}{\hat{\epsilon}_\alpha}$. These supply terms allow us the versatility to better represent composite materials.

Next we assume each of the constituents has the same particle velocity following the shock. That is

$$v_1^- = v_2^- = \cdots = v_k^- = v^- \tag{28}$$

The last equality is obtained from Equation (4). This assumption implies that the constituents may not diffuse and we have from Equation (5)

$$u_\alpha = 0, \qquad \alpha = 1, 2, \ldots, k \tag{29}$$

When no diffusion occurs, the definitions for total pressure, internal energy, and heat flux are greatly simplified. Using Equations (21) and (29) in Equations (7), (8), and (9), we now have

$$P = \sum_\alpha P_\alpha \tag{30}$$

$$\epsilon = \sum_\alpha c_\alpha \epsilon_\alpha \tag{31}$$

$$h = \sum_\alpha h_\alpha \tag{32}$$

Equation (28) also yields certain other simplifications. Equation (25) can now be written as

$$\eta_\alpha = \frac{U}{U - v^-} + \frac{\underset{\sigma}{\hat{c}}{}_\alpha}{\rho_{\alpha_0}(U - v^-)} \tag{33}$$

where η_α is the compression in S_α

$$\eta_\alpha = \rho_\alpha^- / \rho_{\alpha_0} \tag{34}$$

and we have assumed $\rho_{\alpha_0} \neq 0$. Summing Equation (25) over α and using Equation (13), we obtain the following familiar result for the whole mixture.

$$\eta = \frac{U}{U - v^-} \tag{35}$$

where

$$\eta = \rho^- / \rho_0 \tag{36}$$

We conclude, then, that whenever the mass supply, $\underset{\sigma}{\hat{c}}{}_\alpha$, vanishes, the compression, η_α, is independent of α.

$$\eta_\alpha = \eta, \quad \text{for all } \alpha \text{ such that } \underset{\sigma}{\hat{c}}{}_\alpha = 0 \tag{37}$$

That is, each constituent is equally compressed. Of course, the compression, η_α, is defined in terms of mixture density, ρ_α, rather than the crystal